Causal Structure of Multiboundary Wormholes

Anderson S. Misobuchi

University of Texas at Austin

February 14, 2020

Introduction

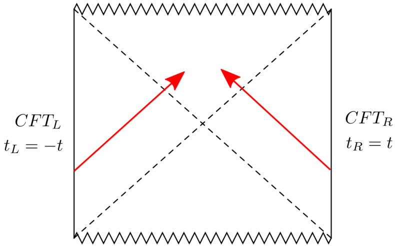
Multiboundary wormholes in 2+1 dimensions

- Natural generalization of eternal BTZ black hole
- Quotients of $AdS_3 \rightarrow$ locally AdS_3
- Traversable multiboundary wormholes?

Eternal BTZ black hole

[Maldacena '01]
Eternal AdS black holes $\leftrightarrow$ Thermofield double state (TFD)

$$H = H_R - H_L$$
$$Z = \text{Tr}\, e^{-\beta H}$$



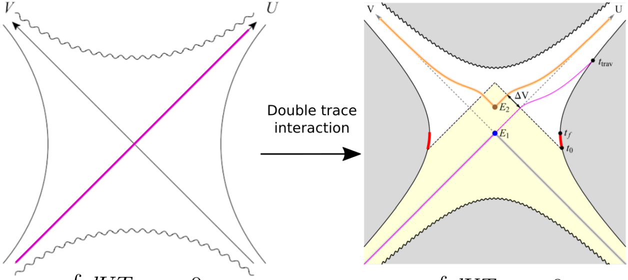
Thermofield double state

$$|TFD\rangle = \frac{1}{\sqrt{Z}} \underbrace{\sum_n e^{-\beta E_n/2} |E_n\rangle_L |E_n\rangle_R}_{\textbf{entangled, non-interacting}}$$

Traversable wormhole via double trace deformation

Double trace interaction [Gao, Jafferis, Wall '16]

$$\delta H(t) = -\int dx\, h(t,x)\mathcal{O}_R(t,x)\mathcal{O}_L(-t,x)$$



$\int dU T_{UU} = 0$ $\int dU T_{UU} < 0$

$\Delta V < 0 \quad =$ "Opening of the wormhole"

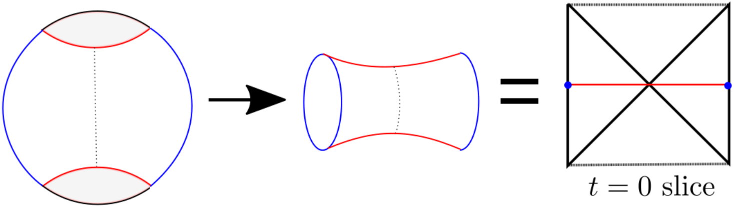
Addition of angular momentum [Caceres, Misobuchi, Xiao '18]

Multiboundary wormholes

- Einstein gravity in 2+1 dimensions with negative cosmological constant: $R_{\mu\nu\rho\sigma} = \Lambda(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$

$$\Rightarrow \text{ Spacetime geometry is locally } AdS_3$$

- BTZ is quotient of AdS_3.

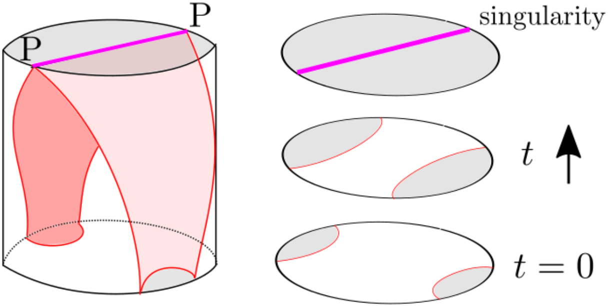


- Multiboundary wormholes are generalizations of eternal BTZ.

$$\mathcal{M} = AdS_3/\Gamma, \quad \Gamma \text{ discrete subgroup}$$

Causal structure of BTZ as a quotient

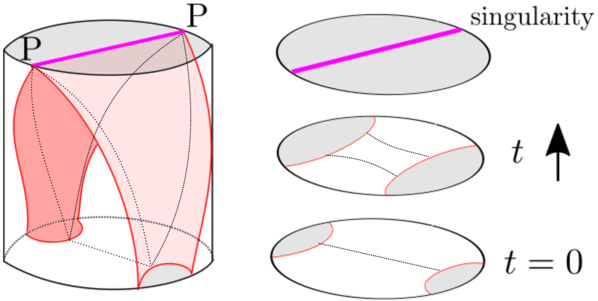
Time dependence: Identified geodesics evolve into geodesic planes [Aminneborg et al '97, Brill '99]



Singularites: Intersection of geodesic planes
Horizon: Backwards light cone of point P

Causal structure of BTZ as a quotient

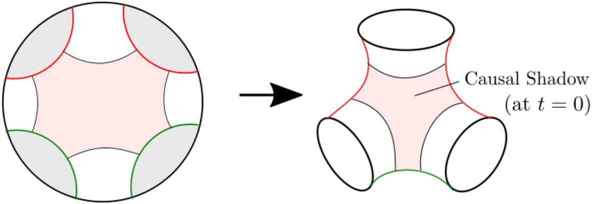
Time dependence: Identified geodesics evolve into geodesic planes [Aminneborg et al '97, Brill '99]



Singularites: Intersection of geodesic planes
Horizon: Backwards light cone of point P

Multiboundary wormhole

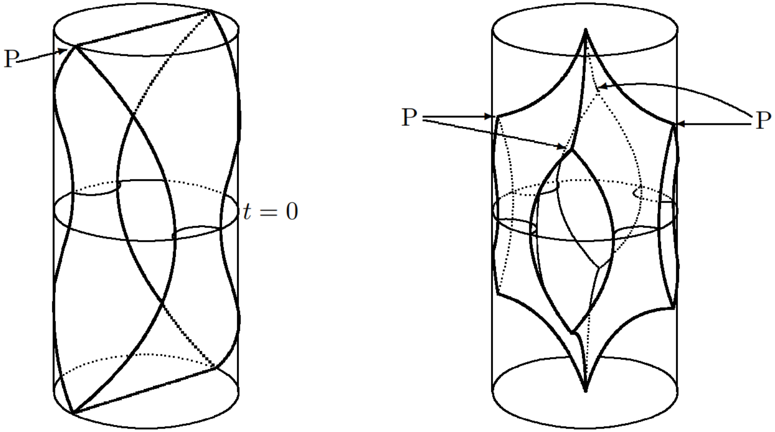
- Three-boundary wormhole (genus zero)



- Curves of minimal length between the identified geodesics characterize the horizons
- **Causal shadow**: Region causally disconnected from all asymptotic boundaries.

Time development of three-boundary wormhole

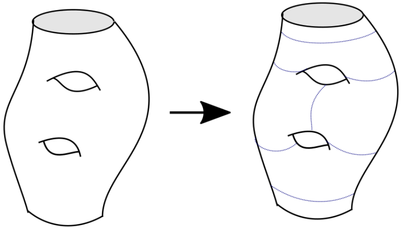
[Aminneborg et al '97, Brill '99]



Fenchel-Nielsen coordinates

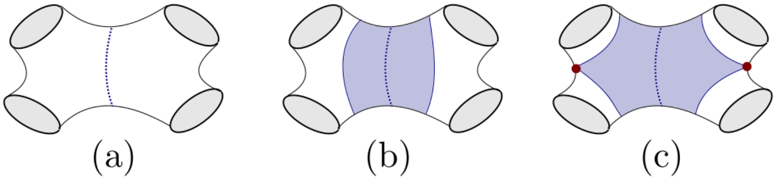
Cut Riemann surface with into pairs of pants [Skenderis, Rees '09]

$$m = \text{ number of boundaries} \qquad g = \text{ genus}$$



- Assign lengths L_i to all edges of every pants
- Assign a twisting parameter t_j for every gluing of two pair of pants

Fenchel-Nielsen coordinates



(a) Pick up a closed geodesic

(b) Thickening geodesic to obtain cylinder

(c) Extend cylinder as far as possible until bounding circles self intersect.

Outer chart coordinates

- **Outer chart**: covers one asymptotic region and part of the interior
- Explicit metric [Skenderis, Rees '09]

$$ds^2 = \frac{\rho^2 + M}{\cosh^2(\sqrt{M}\tau)}(-d\tau^2 + d\phi^2) + \frac{d\rho^2}{\rho^2 + M}$$

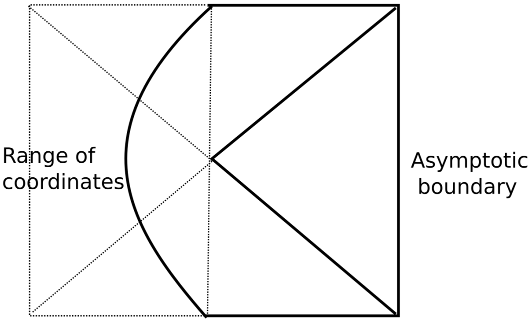
- Horizons lie at the surfaces $\rho = \sqrt{M}|\sinh(\sqrt{M}\tau)|$
- Exterior is exactly BTZ

$$ds^2_{\mathsf{BTZ}} = -(r^2 - M)dt^2 + \frac{dr^2}{r^2 - M} + r^2 d\phi^2$$

Outer chart coordinates

- Singularity $r = 0$ corresponds to $\tau \to \pm\infty$
- Boundaries at $\rho = \pm\infty$

$$\frac{\cosh(\sqrt{M}\tau)\rho}{\sqrt{\rho^2 + M}} > -\sqrt{\frac{C'^2 + C''^2 + 2C'C''C}{C'^2 + C''^2 + C^2 + 2C'C''C - 1}}, \quad C' = \cosh(\pi\sqrt{M'})$$



Transition functions

$$\rho' = -\sqrt{\frac{M'}{M}}\left(\cosh(A)\rho + \sinh(A)\cosh\left(\sqrt{M}(\phi - f)\right)\frac{\sqrt{\rho^2 + M}}{\cosh(\sqrt{M}\tau)}\right)$$

$$\sqrt{M}\tanh(\sqrt{M'}\tau')\sqrt{\rho'^2 + M'} = \sqrt{M'}\tanh(\sqrt{M}\tau)\sqrt{\rho^2 + M}$$

$$e^{2\sqrt{M'}(\phi' - f')} = \frac{\rho\cosh(\sqrt{M}\tau) + \sqrt{\rho^2 + M}\cosh(\sqrt{M}(\phi - f) - b)}{\rho\cosh(\sqrt{M}\tau) + \sqrt{\rho^2 + M}\cosh(\sqrt{M}(\phi - f) + b)}$$

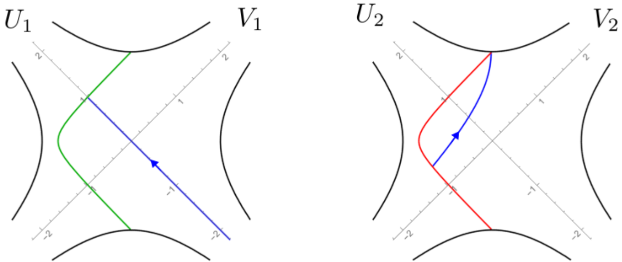
Transition from $1 \to 2$ chart (unprimed to primed) has $f = 0, f' = \pi$.

$$\cosh(A) = \frac{\cosh(\pi\sqrt{M})\cosh(\pi\sqrt{M'}) + \cosh(\pi\sqrt{M''})}{\sinh(\pi\sqrt{M})\sinh(\pi\sqrt{M'})}$$

$$\sinh(A)\sinh(b) = 1$$

Null geodesics in three-boundary wormhole

Null curve starting from past horizon of chart (U_1, V_1)



- Shift in U_2 direction in chart (U_2, V_2)
- Dependence on angular variable

To do

- More complete characterization of causal shadow
- Bulk-to-boundary propagators
- Construction of a traversable three-boundary wormhole
- Extension to more boundaries, non-zero genus