Sparse SYK and traversable wormholes

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Overview

(All-to-all) SYK: Quantum mechanical system of N Majorana fermions χ^j with all-to-all random interactions [Kitaev '15]

$$H = i^{q/2} \sum_{1 \leq j_1 < \ldots < j_q \leq N} \underbrace{J_{j_1 \ldots j_q}}_{\text{Gaussian}} \underbrace{\chi^{j_1} \ldots \chi^{j_q}}_{q\text{-body}}, \qquad \langle \left(J_{j_1 \ldots j_q}\right)^2 \rangle = \frac{(q-1)! J^2}{N^{q-1}}$$

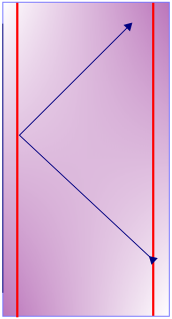
- Analytically solvable at large N
- Emergent conformal symmetry at low energies
- Maximally chaotic $\lambda_L = \frac{2\pi}{\beta}$ [Maldacena, Shenker, Stanford 1503.01409]

Sparse SYK: Modification of SYK model that makes numerical simulations more tractable [Xu, Susskind, Su, Swingle 2008.02303]

Motivation: Understand finite N effects in the quantum mechanical system, and see how gravitational behavior emerges as N increases

Overview

Eternal traversable wormhole with a global AdS_2 geometry can be realized by coupling two copies of SYK in the large N and small coupling limit [Maldacena, Qi 1804.00491]



- Solution can be obtained from JT gravity by adding coupling between boundaries
  [Gao, Jafferis, Wall 1608.05687]
- Traversable for any time
- Same physics can be derived from two coupled SYKs

$$H = H_L^{\text{SYK}} + H_R^{\text{SYK}} + H_{\text{int}}, \quad H_{\text{int}} = i\mu \sum_{j=1}^{N} \chi_L^j \chi_R^j$$

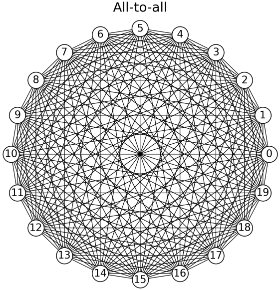
$\rightarrow$ Study two coupled sparse SYKs

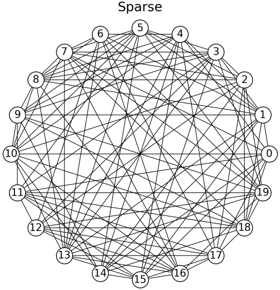
Sparse SYK model

Sparsity: Reduce number of terms in the Hamiltonian summation while preserving original properties, e.g., chaotic behavior
[Xu, Susskind, Su, Swingle 2008.02303]

$$H = i^{q/2} \sum_{1 \le j_1 < \ldots < j_q \le N} J_{j_1 \ldots j_q} x_{j_1 \ldots j_q} \chi^{j_1} \ldots \chi^{j_q},, \quad \text{where} \quad x_{ijkl} = 0 \text{ or } 1$$

Quantum chaos occurs for sparse Hamiltonian with $O(N)$ terms
[Garcia-Garcia et al. 2007.13837]





Sparse SYK model

Hypergraphs: Generalization of a graph where hyperedges can connect more than two vertices

Hamiltonian as Hypergraphs: Majorana fermions are identified with vertices, and each interaction term correspond to a hyperedge connecting q vertices

kq-regular hypergraphs: Every vertex is contained in exactly kq hyperedges.

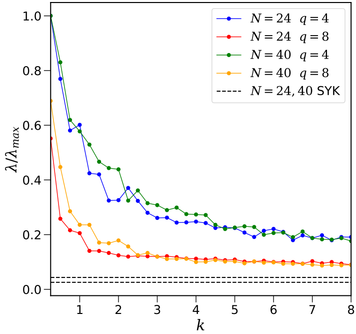
- q indicates that Hamiltonian contains q-body interactions
- k quantifies the degree of sparsity in the Hamiltonian

$\Rightarrow$ Sparse Hamiltonian is a sum of exactly kN terms

Hypergraph expansion

We want a sparse hypergraph that is still highly connected

$$[A]_{ij} = \begin{cases} \#\text{ of hyperedges containing vertices } i \text{ and } j & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$



- Second largest eigenvalue λ $\rightarrow$ Spectral gap
- Indication that $k \gtrsim 1$ has good connectivity
- Other measures of hypergraph expansion: algebraic entropy and vertex expansion

Two coupled SYK model

Properties of the two coupled SYK model
[Maldacena, Qi 1804.00491]

$$H = H_L^{\mathsf{SYK}} + H_R^{\mathsf{SYK}} + i\mu \sum_{j=1}^{N} \chi_L^j \chi_R^j$$

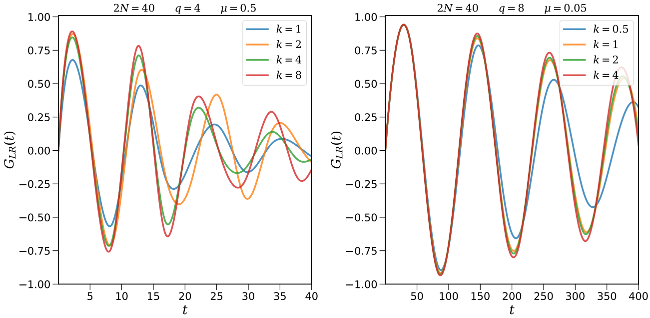
- Ground state $|\Psi_0\rangle$ approximately a TFD state (for some $\beta(\mu)$)
- Energy gap scaling (Derived from large N analysis (gravitational))

$$E_{\mathsf{gap}} \sim \mu^{\frac{1}{2-2/q}} \qquad \text{at weak coupling}$$
$$E_{\mathsf{gap}} \sim \mu \qquad \text{at strong coupling}$$

$\rightarrow$ Characteristic frequency of oscillation of two-point functions

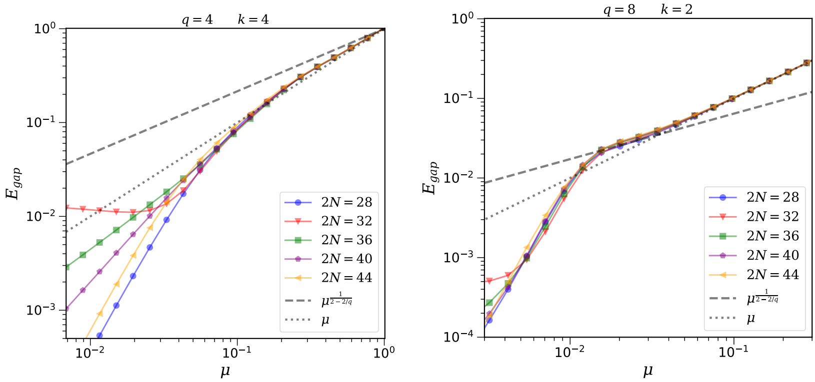
Green's functions

$$G_{ab}(t) = \frac{1}{N}\sum_j 2\mathrm{Re}\langle \chi_a^j(t)\chi_b^j(0)\rangle, \qquad a, b = L, R.$$



- Sparse SYK similar to original SYK using k of order 1
- Larger q allows us to choose smaller k

Energy gap



- $q = 8$ matches scaling expected from large N for some range of couplings
- Finite N effects dominate at small couplings μ

Revival dynamics phenomena

1. Start with ground state $|\Psi_0\rangle$ of the two coupled SYK
2. Create Majorana excitation in Right system

$$|\Psi(t=0)\rangle = \chi_R|\Psi_0\rangle$$

3. Excitation gets scrambled
4. Excitation reassembles and becomes localized in Left system

$$|\Psi(t=t_{\text{rev}})\rangle = \chi_L|\Psi_0\rangle$$

5. Process is repeated with $L \leftrightarrow R \Rightarrow$ 'Revival oscillations'
   [Plugge, Lantagne-Hurtubise, Franz 2003.03914]

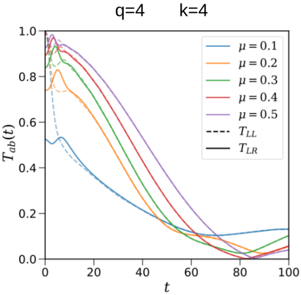
Gravity picture: Perturbation travels through the wormhole

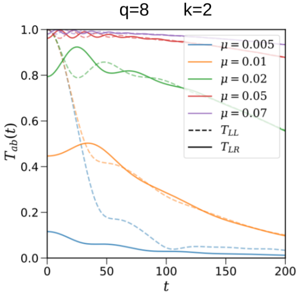
Diagnostic of revivals

Transmission amplitude $\quad T_{ab} = 2|G^{>}_{ab}|$

$$G^{>}_{ab}(t) = -\frac{i}{N}\sum_j \langle \mathcal{T}\chi^j_a(t)\chi^j_b(0)\rangle = \begin{pmatrix} G^{>}_{LL}(t) & G^{>}_{LR}(t) \\ G^{>}_{RL}(t) & G^{>}_{RR}(t) \end{pmatrix}$$

$|T_{ab}(t)|^2$: Probability of recovering χ^j_a at some time t after inserting χ^j_b at $t = 0$.

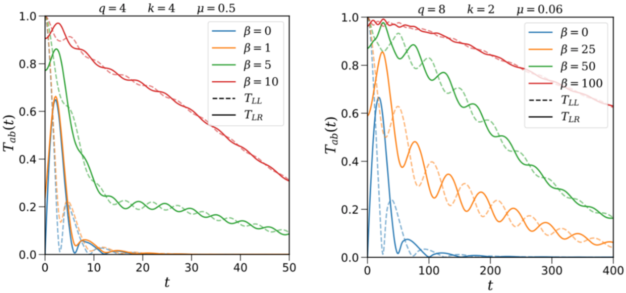




(At zero temperature, $2N = 40$)

Diagnostic of revivals

Increasing temperature: Decreases transmission amplitude but enhances the oscillatory behavior



SYK with $q = 4$ does not follow expected frequency scaling from AdS_2 gravity, but $q = 8$ is compatible for some range

$\rightarrow$ Suggests that finite N effects are supressed by $1/q$

Summary: Sparse SYK is a computationally tractable quantum mechanical system with emergent gravitational behavior and can describe an eternal traversable wormhole

Future directions:

- Out-of-time-order (OTOCs) correlators
- Relation with quantum teleportation protocols
- Quantum complexity

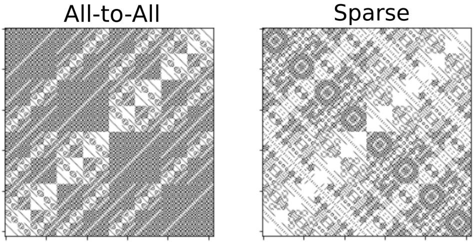
dynamite: a python library that makes use of PETSc and SLEPc. Krylov subspace methods combined with massive parallelization [Github:GregDMeyer/dynamite]

Texas Advanced Computing Center (TACC): Use of computational resources from Stampede2 supercomputer

Numerical methods

SYK maps to $N/2$-qubit system via **Jordan-Wigner transformation**

$$\chi_{2n} = \left(\prod_{j=1}^{n-1} \sigma_j^x\right) \sigma_n^z, \quad \chi_{2n-1} = \left(\prod_{j=1}^{n-1} \sigma_j^x\right) \sigma_n^y, \qquad \{\chi_i, \chi_j\} = 2\delta_{ij}$$



Numerical methods

Krylov subspace

$$\mathcal{K}_m = \mathsf{span}\{|\psi(t)\rangle, H|\psi(t)\rangle, H^2|\psi(t)\rangle, \ldots, H^{m-1}|\psi(t)\rangle\}$$

Get approximation for time evolution

$$e^{-iH\Delta t}|\psi(t)\rangle \simeq V_m e^{-iV_m H V_m \Delta t} e_1$$

Typicality:

$$\langle \chi_a^j(t)\chi_b^j(0)\rangle = \frac{1}{Z}\mathsf{Tr}\left[e^{-\beta H}\chi_a^j(t)\chi_b^j(0)\right] \simeq \frac{\langle\beta|\chi_a^j(t)\chi_b^j(0)|\beta\rangle}{\langle\beta|\beta\rangle}$$

$|\beta\rangle = e^{-\frac{\beta}{2}H}|\psi\rangle$, $\quad|\psi\rangle$ random state