Traversable wormholes from a sparse SYK model

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Overview

Eternal traversable wormhole with a global AdS_2 geometry can be realized by coupling two copies of SYK in the large N and small coupling limit [Maldacena, Qi '18]

Motivation: Understand finite N effects in the quantum mechanical system, and see how gravitational behavior emerges as N increases

Sparse SYK: Modification of SYK model which makes numerical simulations more tractable [Xu, Susskind, Su, Swingle '20]

$\rightarrow$ Study two coupled sparse SYKs

Sachdev-Ye-Kitaev (SYK) model

SYK: Quantum mechanical system of N Majorana fermions with all-to-all random interaction

$$H_{\text{SYK}} = \sum_{1 \le i < j < k < l \le N} \underbrace{J_{ijkl}}_{\text{Gaussian}} \underbrace{\chi_i \chi_j \chi_k \chi_l}_{4-\text{local}}, \qquad \langle J_{ijkl}^2 \rangle = \frac{3! J^2}{N^3}$$

Large N $\rightarrow$ Analytically solvable

- Emergent conformal symmetry at low energies
- Saturation of chaos bound $\lambda_L = \frac{2\pi}{\beta}$

Finite N $\rightarrow$ Numerical simulations

- SYK simulation up to $N = 60$ Majorana fermions using Krylov subspace methods (matrix-free) [Kobrin et al. 2002.05725]
- dynamite: a python library that makes use of PETSc and SLEPc allowing massive parallelization

  [Github:GregDMeyer/dynamite]

Sparse SYK model

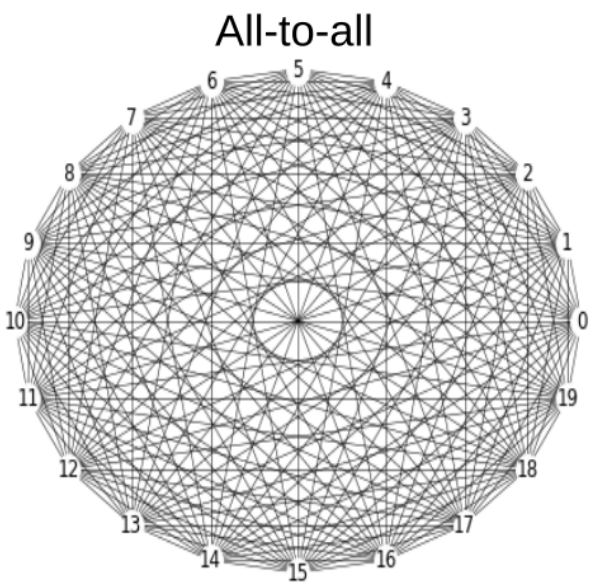
Sparsity: Reduce number of terms in the Hamiltonian summation while preserving original properties, e.g., chaotic behavior

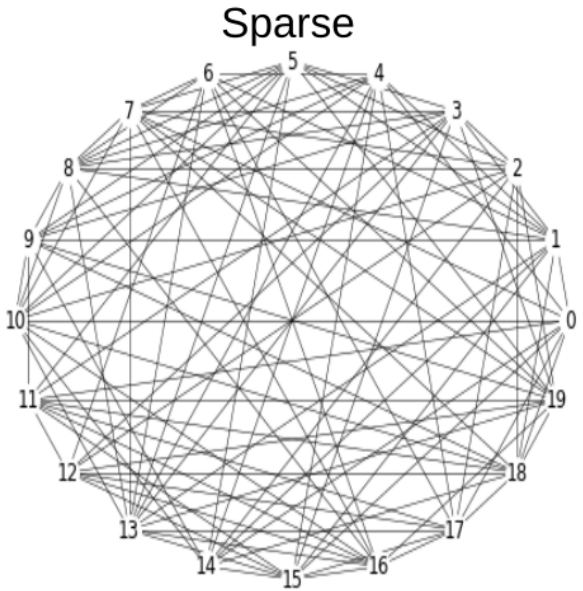
[Xu, Susskind, Su, Swingle 2008.02303]

$$H = \sum_{i<j<k<l} x_{ijkl}\, J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l, \qquad \text{where} \qquad x_{ijkl} = 0 \text{ or } 1$$

Quantum chaos occurs for sparse Hamiltonian with $O(N)$ terms

[Garcia-Garcia et al. 2007.13837]





Sparse SYK model

Hypergraphs: Generalization of a graph where hyperedges can connect more than two vertices

Hamiltonian as Hypergraphs: Majorana fermions are identified with vertices, and each interaction term correspond to a hyperedge connecting $q = 4$ vertices

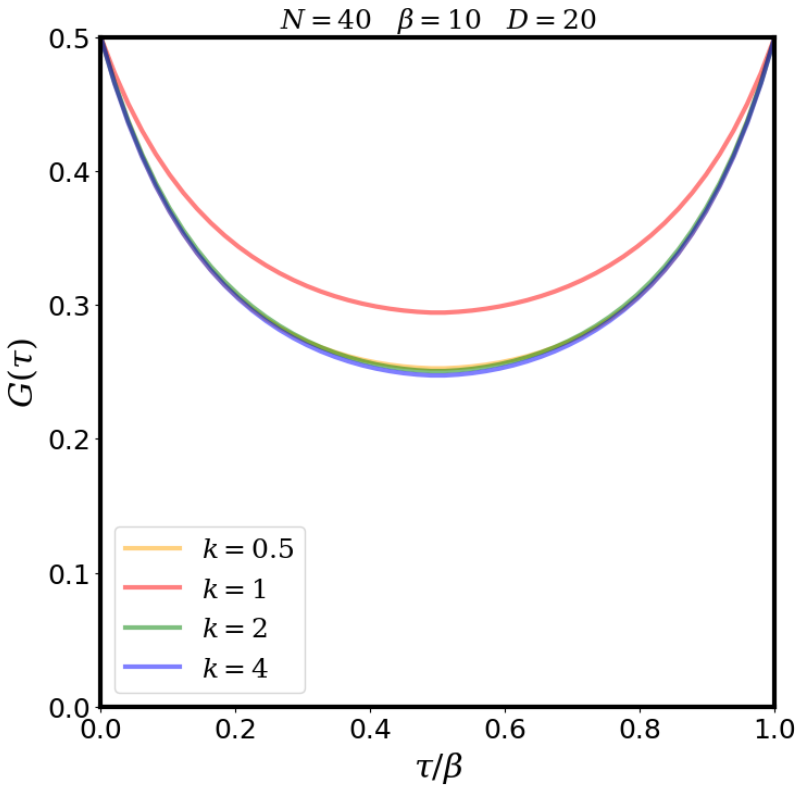
kq-regular hypergraphs: Every vertex is contained in exactly kq-hyperedges.

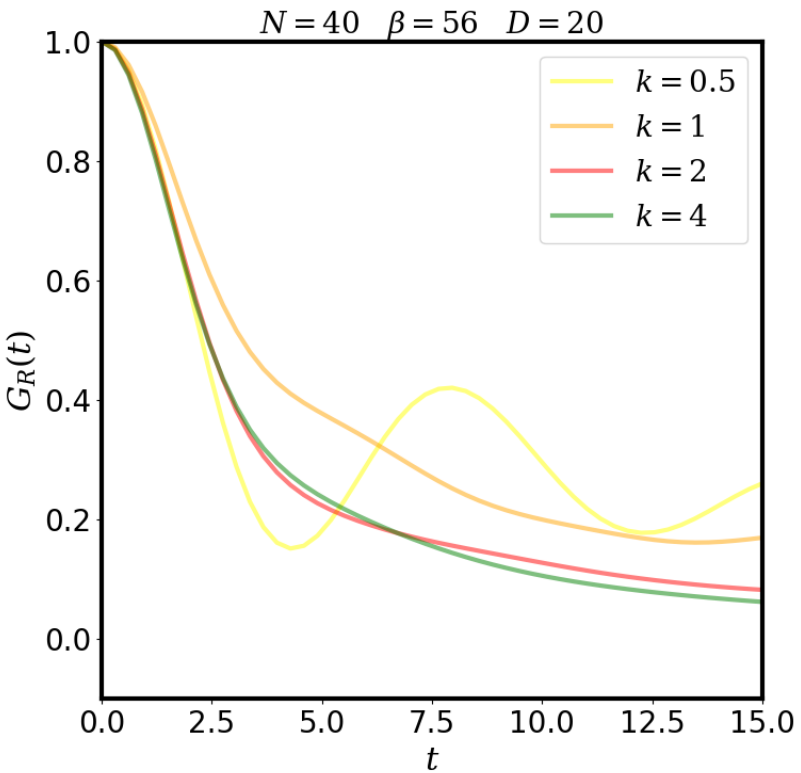
- q indicates that Hamiltonian is q-local
- k quantifies the degree of sparsity in the Hamiltonian

$\Rightarrow$ Sparse Hamiltonian is a sum of exactly kN terms

Example: Green's function

$$G(\tau) = \langle \chi_i(\tau)\chi_i(0)\rangle \qquad G_R(t) = 2\mathrm{Re}\langle \chi_i(t)\chi_i(0)\rangle$$

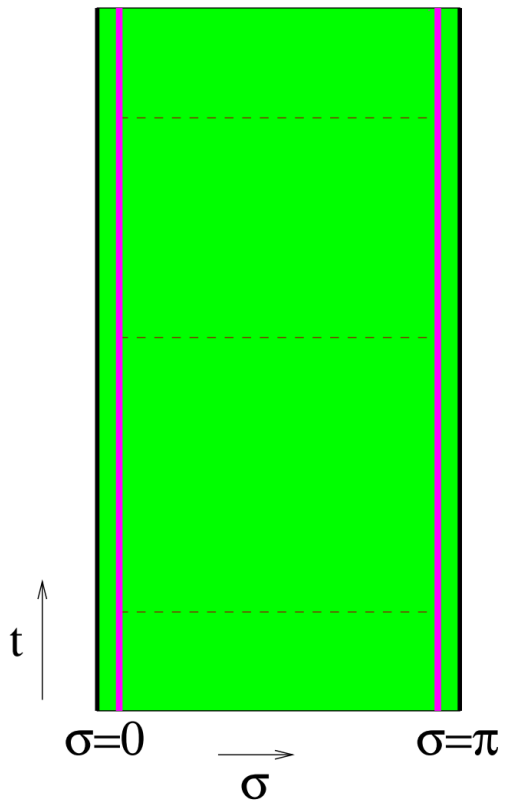




Eternal traversable wormhole

Global AdS_2 has two causally connected boundaries

$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}, \quad \sigma \in [0, \pi]$$



[Maldacena, Qi 1804.00491]

- Would require matter that violates Null Energy Condition
- Can be realized by adding a coupling between the boundaries

$$S_{\text{int}} = g \sum_{i=1}^{N} \int du \mathcal{O}_L^i(u) \mathcal{O}_R^i(u)$$

- Traversable for any time

Revival dynamics phenomena

Two coupled SYK Hamiltonian

$$H = H_L^{\text{SYK}} + H_R^{\text{SYK}} + H_{\text{int}}, \qquad H_{\text{int}} = i\mu \sum_{j=1}^{N} \chi_L^j \chi_R^j,$$

[Plugge, Lantagne-Hurtubise, Franz 2003.03914]

1. Start with ground state $|G\rangle$ of two coupled SYK
2. Create Majorana excitation in Right system

$$|\Psi(t=0)\rangle = \chi_R |G\rangle$$

3. Excitation gets scrambled
4. Excitation reassembles and becomes localized in Left system

$$|\Psi(t=t_{\text{rev}})\rangle = \chi_L |G\rangle$$

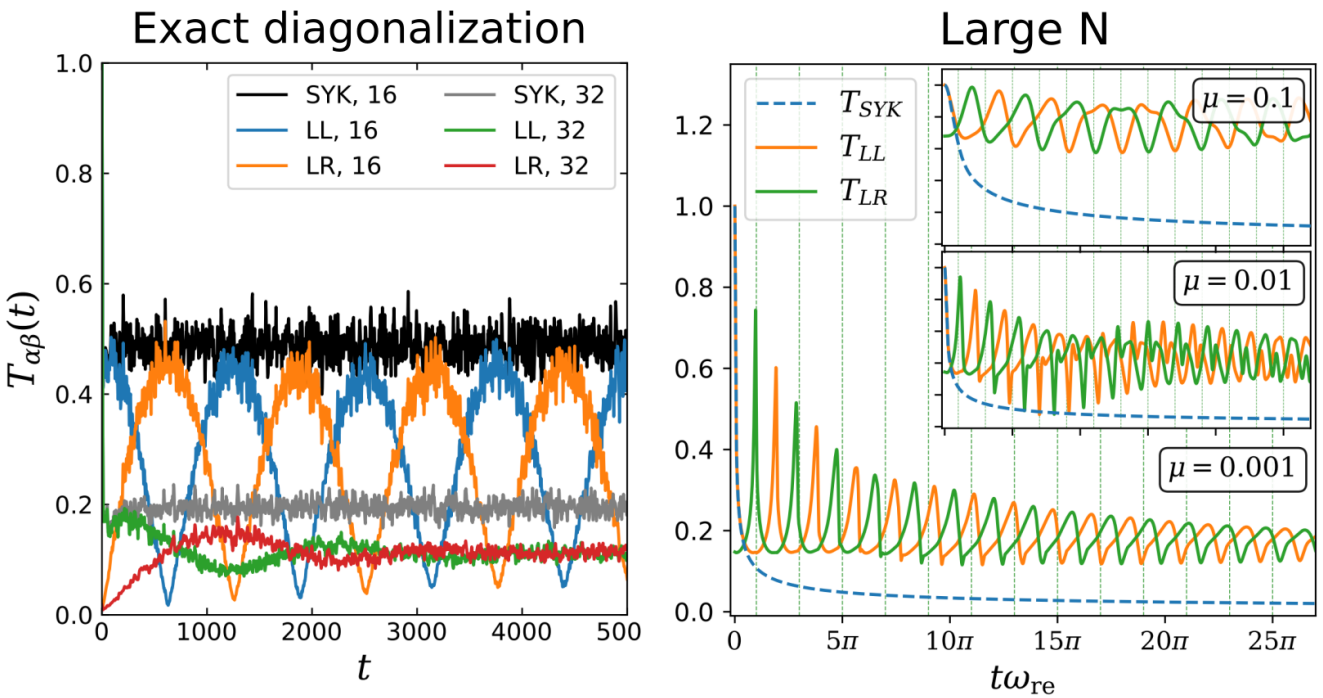
5. Process is repeated with $L \leftrightarrow R \Rightarrow$ 'Revival oscillations'

Revivals in all-to-all SYK

Transmission amplitude $T_{LR}(t) = |\langle \Psi(t)|\Psi_L\rangle|$

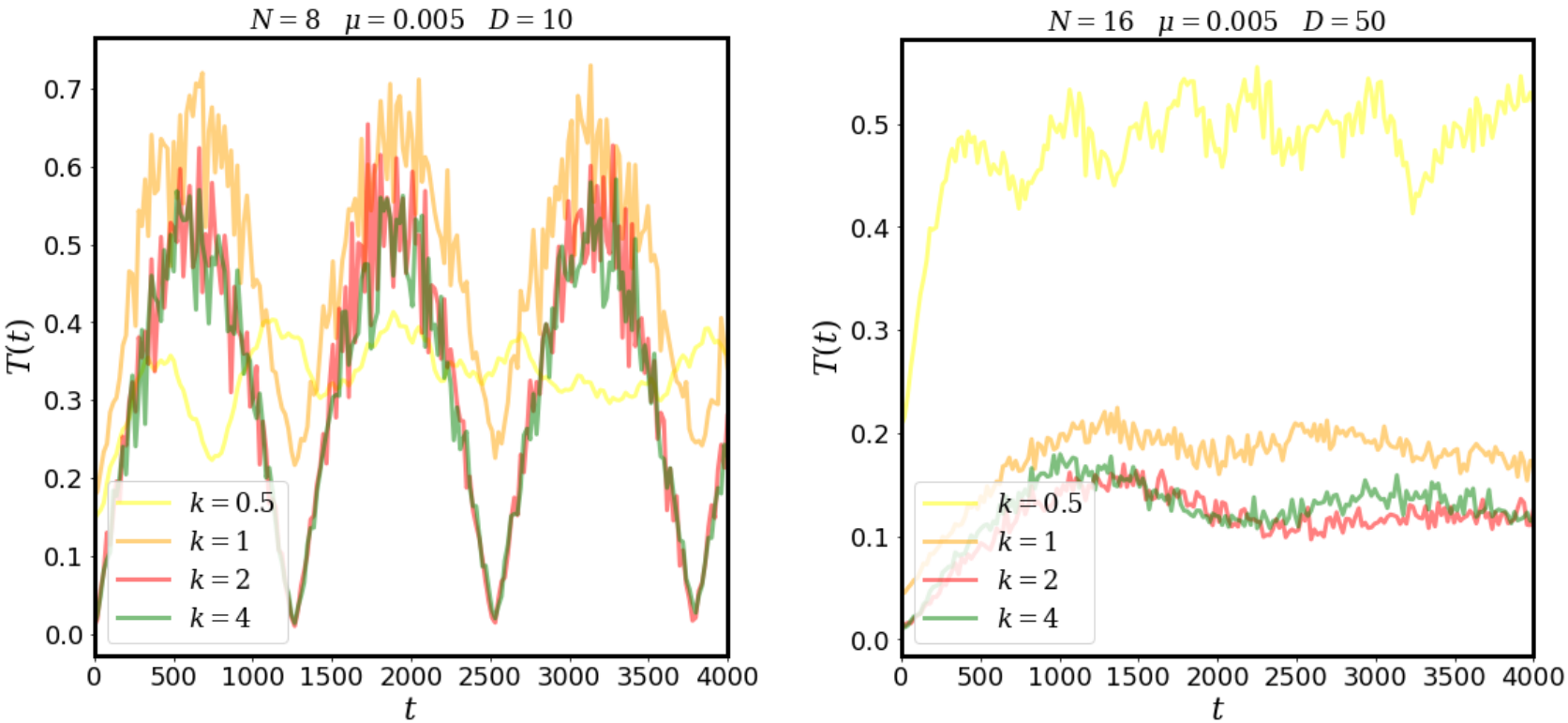
$$|\Psi_R\rangle = \chi_R|G\rangle \qquad |\Psi_L\rangle = \chi_L|G\rangle \qquad |\Psi(t)\rangle = e^{-iHt}\chi_R|G\rangle$$

[Plugge, Lantagne-Hurtubise, Franz 2003.03914]



- Large N: Frequency of oscillations scale as $\omega \sim \mu^{\frac{2}{3}}$
- Finite N: (up to 2N=32) Amplitude decreases as N increases, and frequency does not scale as in the large N limit

Revivals in sparse SYK



(Averaged over D disorder realizations of the random couplings)

- Sparsity level $k = 4$ gives quantitative agreement with numerical results for all-to-all SYK

Summary:

- Two coupled sparse SYK is a numerically tractable quantum system that can be connected to a traversable wormhole geometry

In progress:

- Larger N simulations using TACC supercomputer (Stampede2)
- Consider $q > 4$ local Hamiltonians
- Make connection with large N solutions