Traversable wormholes

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Overview

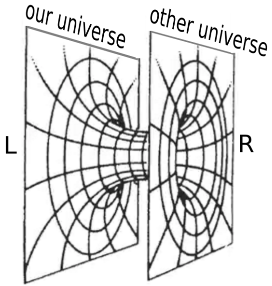
"Traversable wormhole via a double trace deformation"
[Gao, Jafferis and Wall, 1608.05687]

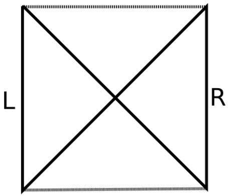
"Diving into traversable wormholes"
[Maldacena, Stanford and Yang, 1704.05333]

Work in progress
[with Elena Caceres and Minglei Xiao]

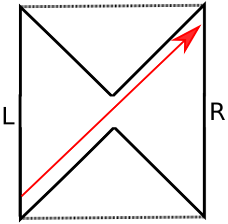
Traversable wormholes

Wormhole: A "throat" connecting two universes or a shortcut connecting two distant places in the same universe





non-traversable wormhole



traversable wormhole

Traversable wormholes: Usually require addition of exotic matter, which violates energy conditions.

Ruling out traversable wormholes

Impose constraints on stress-tensor $T_{\mu\nu}$ in order to render reasonable physical theories

Average Null Energy Condition (ANEC)

$$\int_{-\infty}^{\infty} T_{\mu\nu} k^{\mu} k^{\nu} d\lambda > 0, \quad \forall \text{ null geodesic}$$

k^{μ} null vector, λ affine parameter

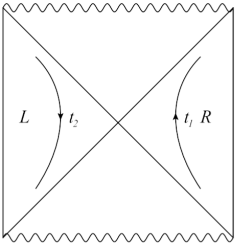
- ANEC in GR $\Rightarrow$ ensures gravity is an attractive force
- At quantum level ANEC can be violated
- For all traversable wormholes: $\Rightarrow$ necessary violation of ANEC [Morris, Thorne, Yurtsever '88]

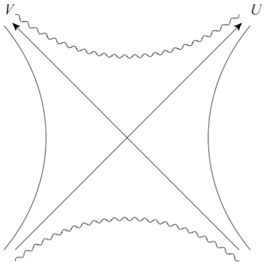
Framework: Embedded in AdS/CFT

BTZ black hole [Banados, Teitelboim, Zanelli '92]

$$ds^2 = -\frac{r^2 - r_h^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2 - r_h^2}dr^2 + r^2 d\phi^2, \quad \phi \sim \phi + 2\pi$$

Kruskal coordinates: $e^{2r_h t} = -\frac{U}{V}, \quad \frac{r}{r_h} = \frac{1-UV}{1+UV}$





Constructing a traversable wormhole

Steps

- Add a scalar field φ dual to an operator $\mathcal{O}$

$$T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \partial_\rho \varphi \partial_\sigma \varphi - \frac{1}{2} g_{\mu\nu} M^2 \varphi^2$$

- Add coupling between left and right sides

$$\delta H(t) = -\int d\phi h(t,\phi) \mathcal{O}_R(t,\phi) \mathcal{O}_L(-t,\phi), \quad h(t,\phi) = \begin{cases} h\left(\frac{2\pi}{\beta}\right)^{2-2\Delta} & , t \geq t_0 \\ 0 & , \quad t < t_0 \end{cases}$$

- Evaluate leading order contribution of the interaction to the integral of the T_{UU} component
- Choose appropriate sign of the coupling in order to the integral be negative and wormhole become traversable

Constructing a traversable wormhole

Evaluation of $\int dU\, T_{UU}$

- Consider correlation function $\langle \varphi_R^H(t,r,\phi)\varphi_R^H(t',r',\phi')\rangle$
- First order contribution in h

$$G_h = i\int_{t_0}^{t} dt_1 h(t_1) K_\Delta(t'+t_1-i\beta/2)[K_\Delta(t-t_1-i\epsilon)-K_\Delta(t-t_1+i\epsilon)]+(t\leftrightarrow t')$$

- bulk-to-boundary propagator

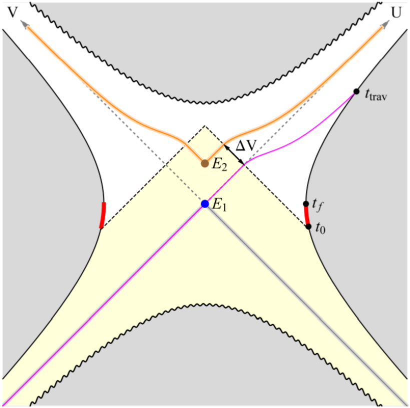
$$K_\Delta(r,t,\phi;0,0) = \frac{r_h^\Delta}{2^{\Delta+1}\pi}\left(-\frac{(r^2-r_h^2)^{1/2}}{r_h}\cosh r_h t + \frac{r}{r_h}\cosh r_h\phi\right)^{-\Delta}$$

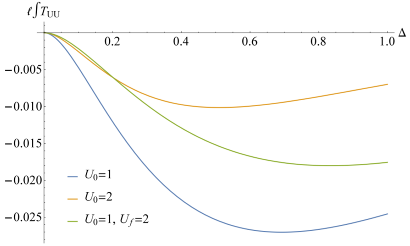
- T_{UU} component of stress tensor obtained by point splitting

$$T_{UU} = \lim_{U'\to U} \partial_U \partial_{U'} G_h$$

Constructing a traversable wormhole

Integration of T_{UU} is negative with appropriate choice of sign of the coupling h.





Linearized Einstein equation

$$V(U) \sim \int dU T_{UU}$$

Negative null energy $\Rightarrow$ Traversability

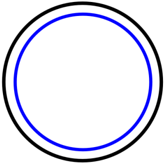
$$\Delta V \sim \frac{h\, G_N}{R^{D-2}}$$

Maldacena, Stanford and Yang

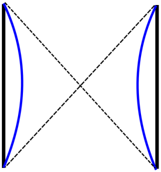
Dilaton gravity in 2D. Jackiw-Teiteboim theory

$$I_{JT} = \frac{1}{16\pi G_N}\left[\int d^2x \Phi \sqrt{g}(R+2) + 2\int_{bdy} \Phi_b K\right]$$

- Equation of motion for Φ: $R+2=0 \Rightarrow$ Rigid AdS_2 geometry
- Dynamics is encoded in the boundary



Euclidean AdS$_2$



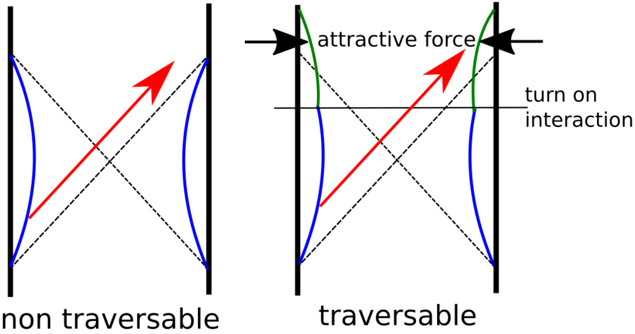
Lorentzian AdS$_2$ black hole (Rindler)

Blue lines: boundary cutoffs

Maldacena, Stanford and Yang

$$e^{i\tilde{g}V} = e^{i\tilde{g}\mathcal{O}_L(0)\mathcal{O}_R(0)}.$$

- Boundary trajectories interpreted as moving charged particles [Kitaev '16]
- Interaction is an attractive force for $g > 0$



Work in progress

- How much can we open the wormhole?
  - Traversability in different geometries
  - Study dependence on charge and angular momentum

Rotating BTZ

Metric:

$$ds^2 = -f^2(r)dt^2 + \frac{dr^2}{f^(r)} + r^2\left[N^\phi(r)dt + d\phi\right]^2,$$

$$f^2(r) = -M + \left(\frac{r}{\ell}\right)^2 + \frac{J^2}{4r^2}, \quad N^\phi(r) = \frac{J}{2}\frac{r^2 - r_+^2}{r^2 r_+^2}.$$

$$r_\pm = \ell\sqrt{\frac{M}{2}\left(1 \pm \sqrt{1 - \left(\frac{J}{M\ell}\right)^2}\right)},$$

Work in progress

Extremal limit

- Spectrum of SYK obtained from 3D near horizon extremal black holes [Das, Jevicki, Suzuki '17]

$$ds^2 = \frac{1}{z^2}(-dt^2 + dz^2) + \left(1 + \frac{a}{z}\right)^2 dy^2$$

- Extremal limit of BTZ: $r_+ \to r_- \Rightarrow$ Possible connection to SYK model

Work in progress

Deformations of AdS_2

- Include a Φ^2 potential

$$I = \frac{1}{16\pi G_N}\left[\int d^2x\sqrt{g}\left[\Phi(R+2)+\alpha\Phi^2\right]+2\int_{bdy}\Phi_b K\right]$$

- $\alpha \sim 1/N$
- Geometry will be no longer AdS_2
- Connection with non-local term that appear in SYK model [Josephine Suh, Kitaev]

Summary

- Adding a coupling between the two copies of the thermofield double yields the wormhole traversable
- Consistent with energy conditions
- Information passing through the wormhole
- Explore traversability in different scenarios. E.g.: rotating BTZ, deformations of AdS_2