Entanglement & Causal Wedge in Gauss-Bonnet gravity

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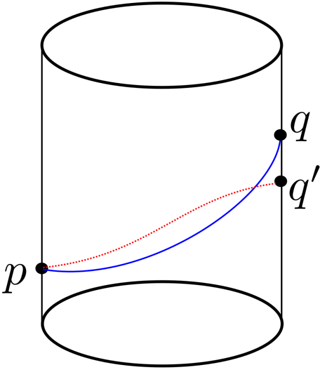
Boundary causality

In holography, bulk dynamics must respect boundary causality



p–q null-related through the boundary

p–q' no causal path through the bulk, q' in the past of q

$\Rightarrow$ No 'shortcut' through the bulk

Boundary causality

Gao-Wald theorem (2000)

In asymptotically AdS spacetimes that obey the Averaged Null Energy Condition (ANEC)

$$\int_{-\infty}^{\infty} du T_{uu} \geq 0$$

there is no causality violating shortcut through the bulk.

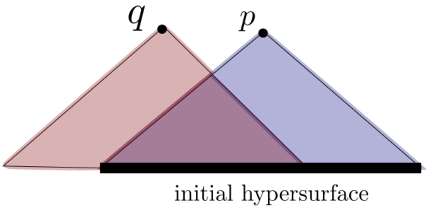
- ANEC is sufficient but not necessary to preserve boundary causality [Engelhardt, Fischetti '16]
- Assumes that causal structure is dictated by null geodesics

Boundary causality

Superluminal modes: Gravitons with faster than light propagation are allowed in higher derivative theories of gravity (e.g. Lovelock)
$\Rightarrow$ Causal structure not governed by null geodesics

Method of characteristics

Characteristic surface: Hypersurface beyond which evolution is not unique. Tangent to characteristic surface gives fastest propagation

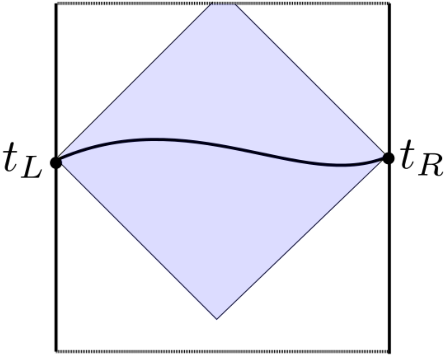


Einstein gravity: characteristic surfaces are null $\Rightarrow$ fastest propagation is speed of light

Implications of superluminal modes

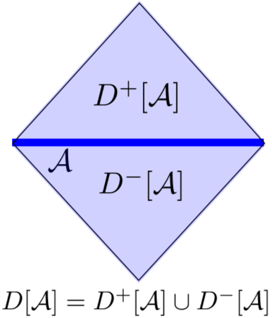
Wheeler-DeWitt patch

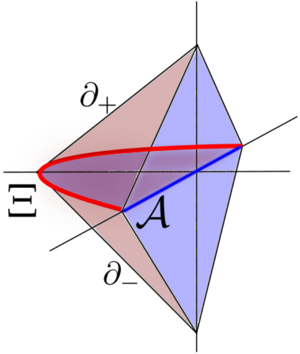
Complexity = Action on the Wheeler-DeWitt patch



Implications of superluminal modes

Causal Wedge





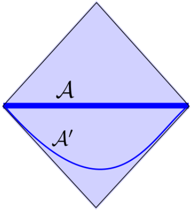
$D[\mathcal{A}]$: Domain of dependence of boundary region $\mathcal{A}$

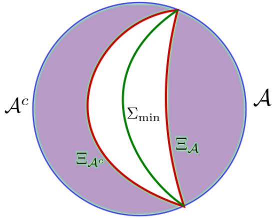
$\partial_\pm$: Surfaces defined by fastest propagation in the bulk

Ξ : Causal information surface

Implications of superluminal modes

Holographic entanglement entropy: $S_{\mathcal{A}} = \dfrac{\text{Area}(\Sigma_{\text{min}})}{4G_N}$





- Wedge observable: $S_{\mathcal{A}} = S_{\mathcal{A}'}$ $\quad$ ($\rho_{\mathcal{A}}, \rho_{\mathcal{A}'}$ unitarily related)
- Perturbation to the Hamiltonian with support entirely inside $D[\mathcal{A}]$ cannot affect entanglement
  $\Rightarrow$Minimal surface Σ_{min} lies in the 'causal shadow'
- NEC is a sufficient condition

[Headrick, Hubeny, Lawrence, Rangamani '14]

Gauss-Bonnet gravity

$$I = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \frac{\lambda L^2}{2}(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})\right)$$

AdS-black hole with spherical horizon

$$ds^2 = -\frac{f(r)}{f_\infty}dt^2 + \frac{dr^2}{f(r)} + r^2(d\phi^2 + \sin^2\phi\, d\Omega_2^2),$$

$$f(r) = r^2\left[\frac{L^2}{r^2} + \frac{1}{2\lambda}\left(1 - \sqrt{1 - 4\lambda + 4\lambda\frac{\mu}{r^4}}\right)\right],$$

$$f_\infty = \frac{1 - \sqrt{1-4\lambda}}{2\lambda}, \quad \mu = r_h^4 + r_h^2L^2 + \lambda L^4.$$

Effective metric

Effective metric for tensor, vector and scalar perturbations [Realla, Tanahashi, Way '14]

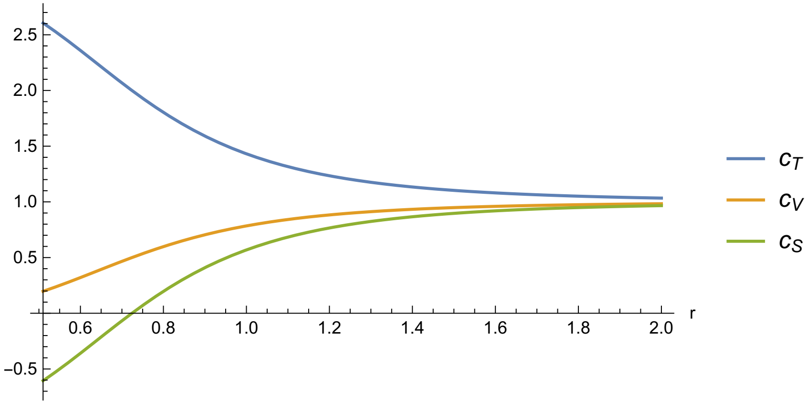
$$ds^2 = -\frac{f(r)}{f_\infty}dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{c_A(r)}d\Omega_3^2$$

$$\begin{cases} c_T(r) = -2A(r) + 3 \\ c_V(r) = A(r) \\ c_S(r) = 2A(r) - 1 \end{cases} \qquad A(r) = \frac{(1-2\lambda)r^4}{2\lambda\mu + (1-2\lambda)r^4}$$

Propagation of perturbations is described by null geodesics in effective metric

$$\dot{t} = \frac{f_\infty}{f(r)}, \qquad \dot{\phi} = \frac{\ell c_A(r)}{r^2}, \qquad \dot{r} = \pm\sqrt{f_\infty - \frac{\ell^2 c_A(r) f(r)}{r^2}}$$

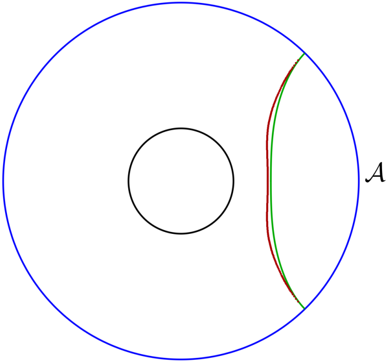
Effective metric



$\lambda > 0$

Entanglement Wedge vs Causal Wedge

$\lambda = 0.15, r_h = 0.5, \phi_{\mathcal{A}} = 0.8$



– Causal wedge
– Entanglement minimal surface

$$S_{\mathcal{A}} = \frac{1}{4G_N} \int_\Sigma d^3x \sqrt{h}(1 + \lambda L^2 R) + \frac{\lambda L^2}{2G_N} \int_{\partial\Sigma} d^2x \sqrt{\tilde{h}} K$$

[Boer, Kulaxizi, Parnachev] [Hung, Myers, Smolkin '11]

Summary

- Higher curvature theories of gravity such as Gauss-Bonnet have superluminal modes that we can use to investigate boundary causality
- Holographic entanglement entropy can be used to test causality violation
- Generalization of Gao-Wald theorem
- More general higher curvature theories
- Holographic complexity