Rotating traversable wormholes in AdS

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October 20, 2018

Traversable wormholes

Traversable wormholes: Necessary violation of ANEC
[Morris, Thorne, Yurtsever '88]

Averaged Null Energy Condition (ANEC)

$$\int_{-\infty}^{\infty} T_{\mu\nu} k^\mu k^\nu d\lambda > 0, \quad \forall \text{ null geodesic}$$

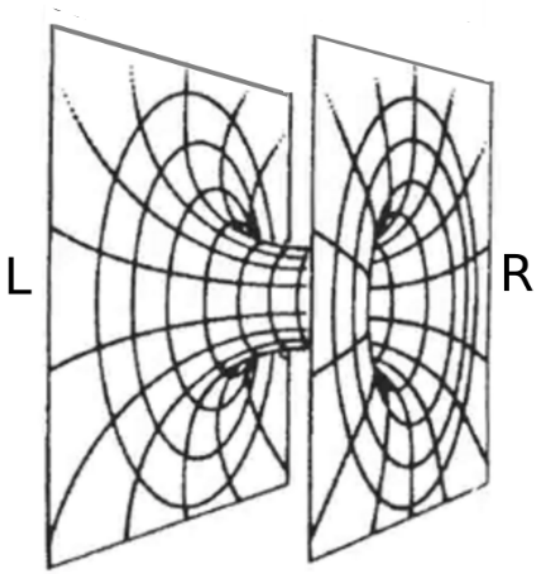
k^μ null vector, λ affine parameter

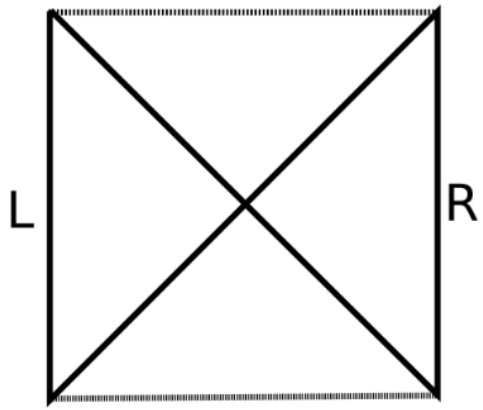
At quantum level, ANEC is expected to hold along achronal geodesics [Graham, Olum '07]

(**Achronal:** No two points can be connected by a timelike curve)

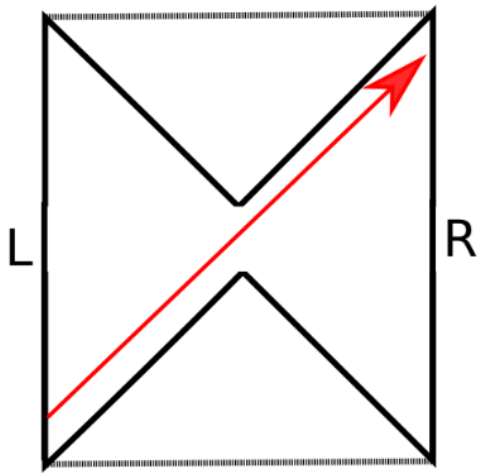
Traversable wormholes

- **Basic idea:** Addition of interaction between the two sides of a wormhole makes it traversable [Gao, Jafferis, Wall '16]
- Construction embedded in AdS/CFT framework
- **Holographic interpretation:** Quantum teleportation





non-traversable wormhole

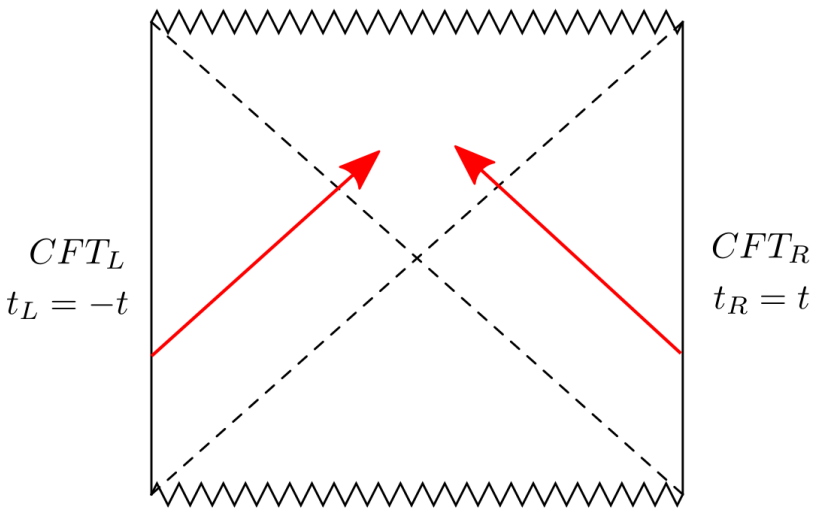


traversable wormhole

Eternal AdS black holes

[Maldacena '01]
Eternal AdS black holes $\leftrightarrow$ Thermofield double state (TFD)

$$H = H_R - H_L$$
$$Z = \mathrm{Tr}\, e^{-\beta H}$$



Thermofield double state

$$|TFD\rangle = \frac{1}{\sqrt{Z}} \underbrace{\sum_n e^{-\beta E_n/2} |E_n\rangle_L |E_n\rangle_R}_{\textbf{entangled, non-interacting}}$$

Traversable wormhole via double trace deformation

Double trace interaction [Gao, Jafferis, Wall '16]

$$\delta H(t) = -\int dx\, h(t,x)\mathcal{O}_R(t,x)\mathcal{O}_L(-t,x)$$

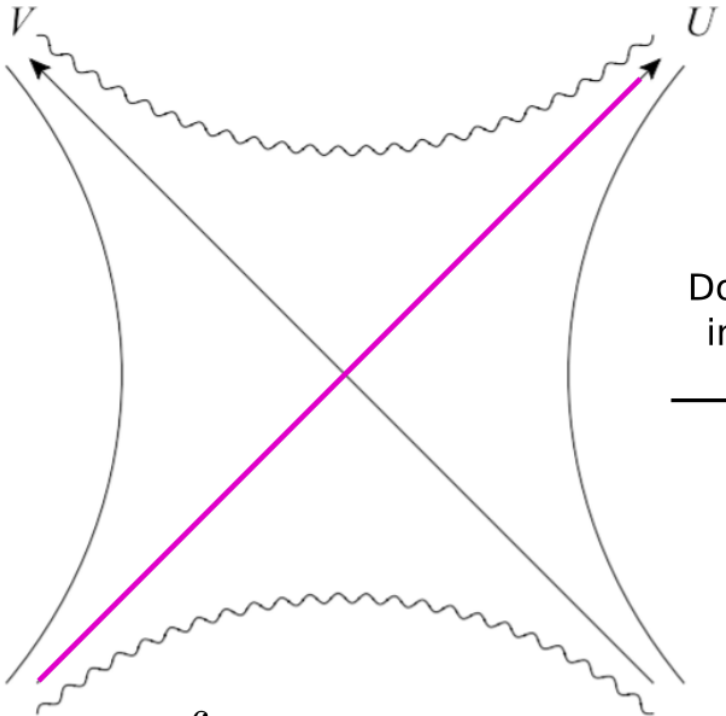
Coupling

$$h(t,x) = \begin{cases} h\,(2\pi/\beta)^{2-2\Delta}, & t_0 \le t \le t_f \\ 0 \qquad\qquad , & \text{otherwise.} \end{cases}$$

$\mathcal{O}_{L/R}$ are dual to a bulk scalar field Φ with associated stress tensor

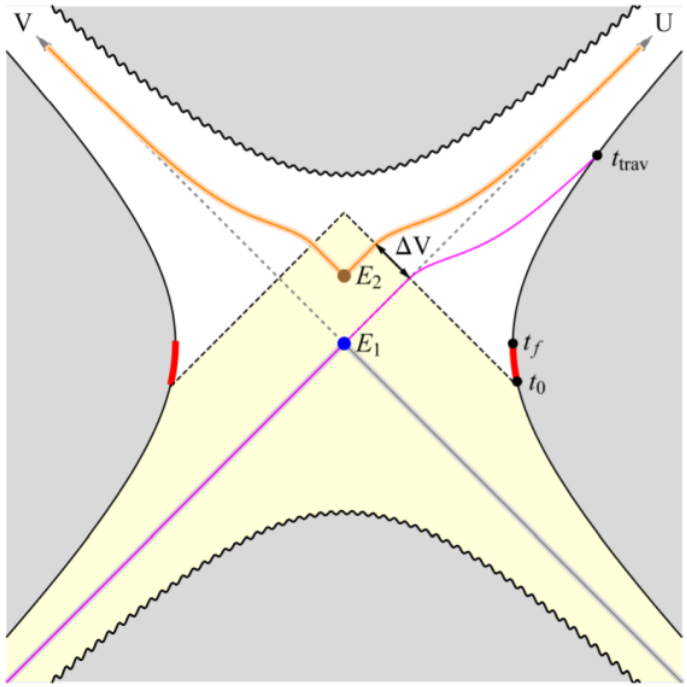
$$T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\Phi\partial_\sigma\Phi - \frac{1}{2}g_{\mu\nu}m^2\Phi^2$$

Traversable wormhole via double trace deformation



$$\int dU T_{UU} = 0$$

Double trace interaction



$$\int dU T_{UU} < 0$$

$\Delta V < 0$ "Opening of the wormhole"

Rotating BTZ black hole

Solution of $2+1$ Einstein gravity with negative cosmological constant [Bañados, Teitelboim, Zanelli '92]

$$ds^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2 r^2}dt^2 + \frac{\ell^2 r^2}{(r^2-r_+^2)(r^2-r_-^2)}dr^2 + r^2\left(\mathcal{N}(r)dt + dx\right)^2$$

$$\mathcal{N}(r) = \frac{r_-}{2r_+}\frac{r^2-r_+^2}{\ell r^2}, \qquad x \sim x + 2\pi.$$

Thermodynamics

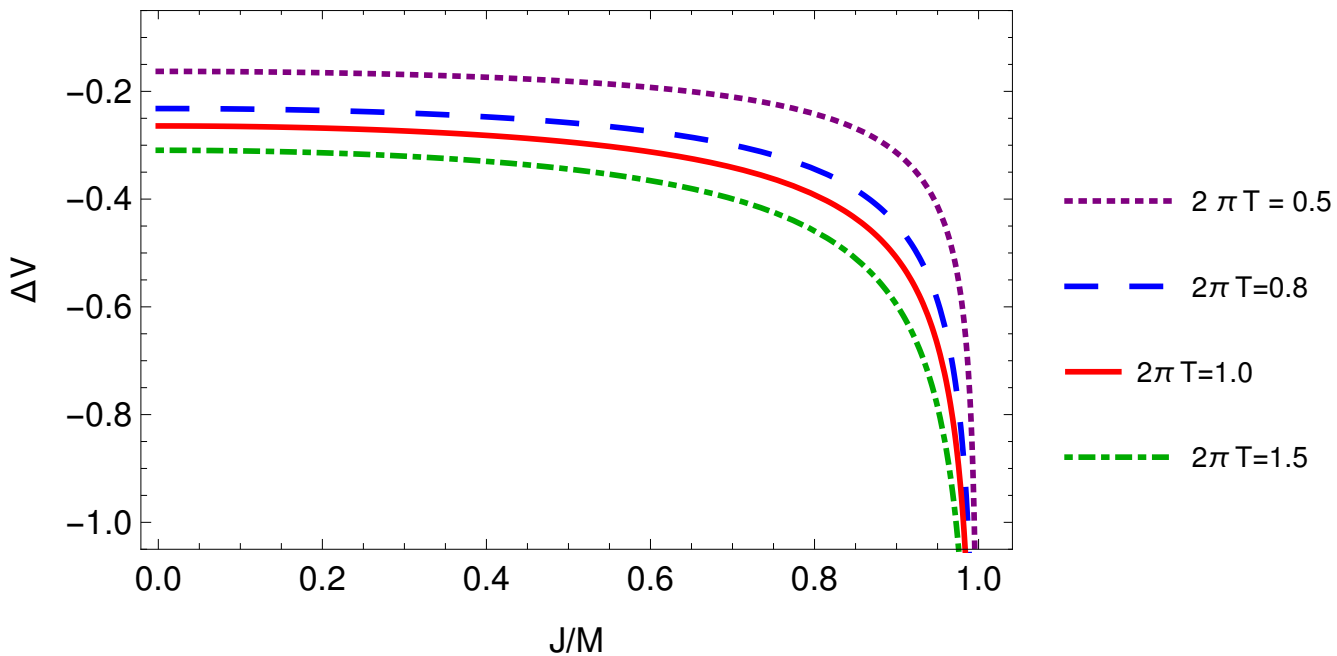
$$M = \frac{r_+^2 + r_-^2}{8G_N\ell^2}, \quad J = \frac{r_+ r_-}{4G_N\ell}, \quad T = \frac{1}{\beta} = \frac{r_+^2 - r_-^2}{\ell^2 2\pi r_+}$$

Opening of wormhole

Linearized Einstein equation

$$\Delta V = \frac{1}{2}\left(\frac{r_+ - r_-}{r_+ + r_-}\right)^{-\frac{r_-}{r_+}} 8\pi G_N \int_{-\infty}^{\infty} dU T_{UU}$$

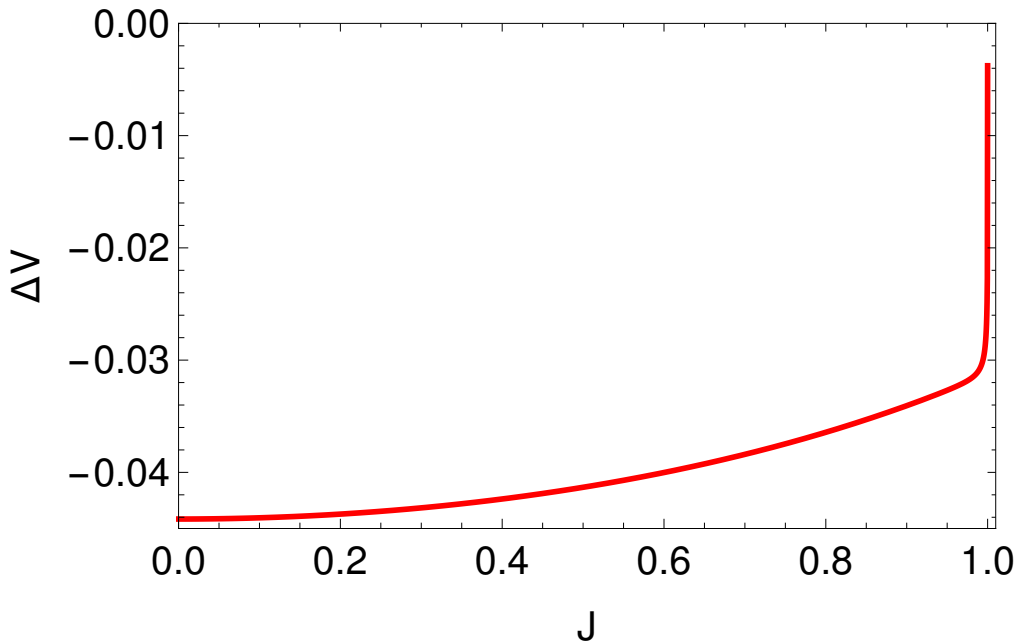
Negative ANE $\Rightarrow$ Traversability



Extremal limit

$$M = \frac{r_+^2 + r_-^2}{8G_N}, \quad J = \frac{r_+ r_-}{4G_N}, \quad T = \frac{r_+^2 - r_-^2}{2\pi r_+}$$

Extremal limit: $r_+ \to r_-, \quad T \to 0, \quad J \to M$

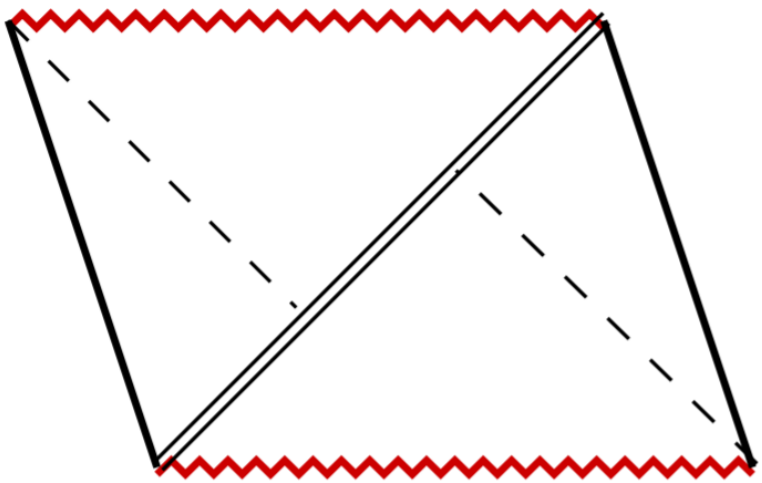


Fixed $M = 1$

$\Rightarrow$ Wormhole closes in extremal limit

Information transfer

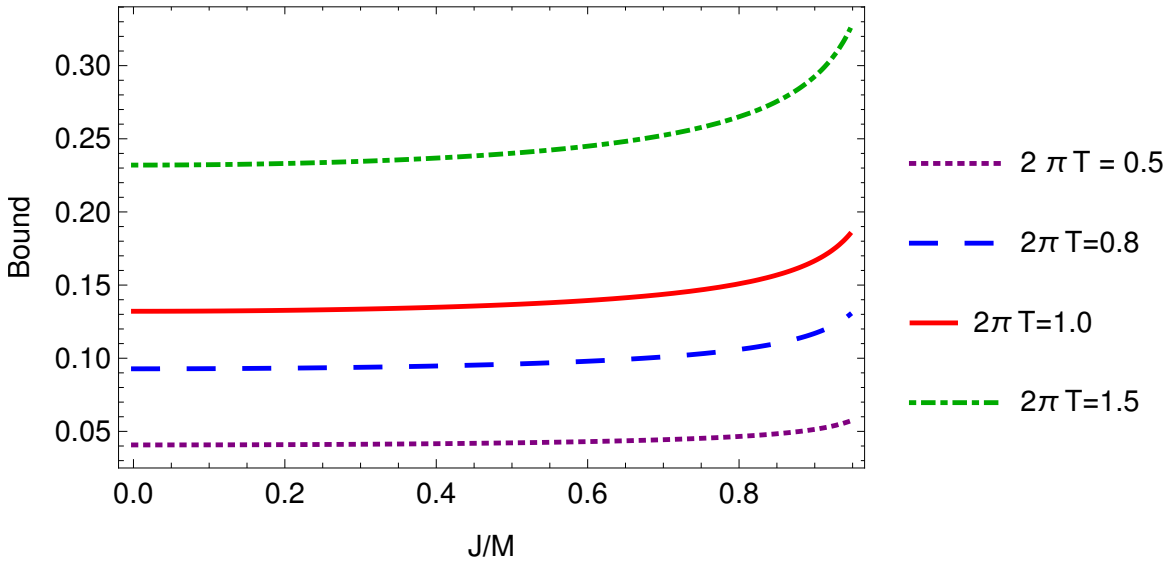
- Backreaction of qubits makes the wormhole longer
  $\Rightarrow$ Limit on the amount of information we can send
- Backreaction characterized by shockwave geometries
  [Shenker, Stanford '13]

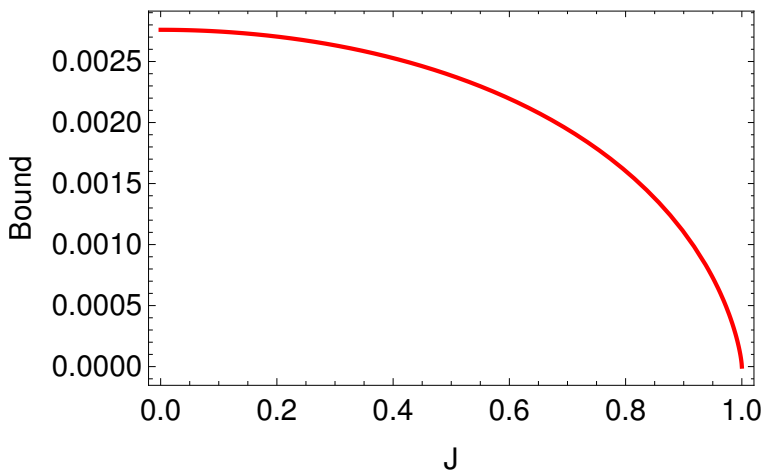


Bound on information transfer

$$N_{\text{send}} \lesssim r_+ \left| \int dx dU T_{UU} \right|$$

Bound on information transfer





Fixed $M = 1$

Conclusions

Summary

- Interaction between the two boundaries violates ANEC and renders the wormhole traversable
- Rotation increases the size of the wormhole ΔV at fixed T
- Bound on information transfer related to the ANE

Extensions

- Eternal traversable wormholes
- Higher dimensional wormholes
- More precise quantum information interpretation