Holographic renormalization and anisotropic black branes in higher curvature gravity

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Introduction

- The AdS/CFT correspondence [1] represents a remarkable tool in the study of the strongly coupled gauge theories which can be mapped to a dual, weakly coupled gravitational description.
- The correspondence is best understood in the limit in which both N and λ, the rank of the gauge group and the 't Hooft coupling of the gauge theory, respectively, are infinite. Investigating departures from this limit implies introducing α' and loop corrections for the string.
- Accounting for higher curvature interactions allows one to begin to consider finite λ and finite N corrections. For example, the leading finite coupling corrections to type IIB supergravity arise as stringy corrections with schematic form $\alpha'^3 R^4$.
- In this work [2] we consider five-dimensional AdS-axion-dilaton gravity with the inclusion of the Gauss-Bonnet (GB) curvature term, given by
$$\mathcal{L}_{GB} = R^2 - 4R_{mn}R^{mn} + R_{mnrs}R^{mnrs}\,.$$
Our aim is to gain some qualitative understanding of the higher curvature effects and, perhaps, uncover some universal properties.
- One application of this study is modeling higher curvature effects on a dual gauge theory anisotropic plasma.

Action and solution

- We consider five-dimensional gravity with negative cosmological constant coupled to an axion-dilaton system and with the inclusion of the Gauss-Bonnet term:
$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\left[R + \frac{12}{\ell^2} - \frac{1}{2}(\partial\phi)^2 - \frac{e^{2\phi}}{2}(\partial\chi)^2 + \frac{\ell^2}{2}\lambda_{GB}\mathcal{L}_{GB}\right].$$
The scalar fields ϕ and χ are the dilaton and axion, respectively, λ_{GB} is the (dimensionless) Gauss-Bonnet coupling and ℓ is a parameter with dimensions of length.
- A solution displaying a spatial anisotropy in the z-direction can be obtained by choosing $\chi = az$, where a is the anisotropy parameter. The z-direction is identified with the 'beam direction' in a heavy ion collision experiment in the boundary theory. For the metric we consider the following Ansatz
$$ds^2 = \frac{1}{u^2}\left(-FB\,dt^2 + dx^2 + dy^2 + H\,dz^2 + \frac{du^2}{F}\right).$$
- Equations of motion can be solved exactly in λ_{GB} and up to order $O(a^2)$ assuming
$$\phi(u) = a^2\phi_2(u) + O(a^4)\,, \qquad F(u) = F_0(u) + a^2F_2(u) + O(a^4)\,,$$
$$B(u) = B_0\left(1 + a^2B_2(u) + O(a^4)\right)\,, \quad H(u) = 1 + a^2H_2(u) + O(a^4)\,,$$
where
$$B_0 = \frac{1}{2}\left(1 + \sqrt{1-4\lambda_{GB}}\right) \quad \text{and} \quad F_0(u) = \frac{1}{2\lambda_{GB}}\left(1 - \sqrt{1 - 4\lambda_{GB}\left(1 - \frac{u^4}{u_H^4}\right)}\right).$$
- Our solution is static, regular everywhere on and outside the horizon u_H, and asymptotically AdS. This solution is the GB-corrected equivalent of the geometry discovered in [3].

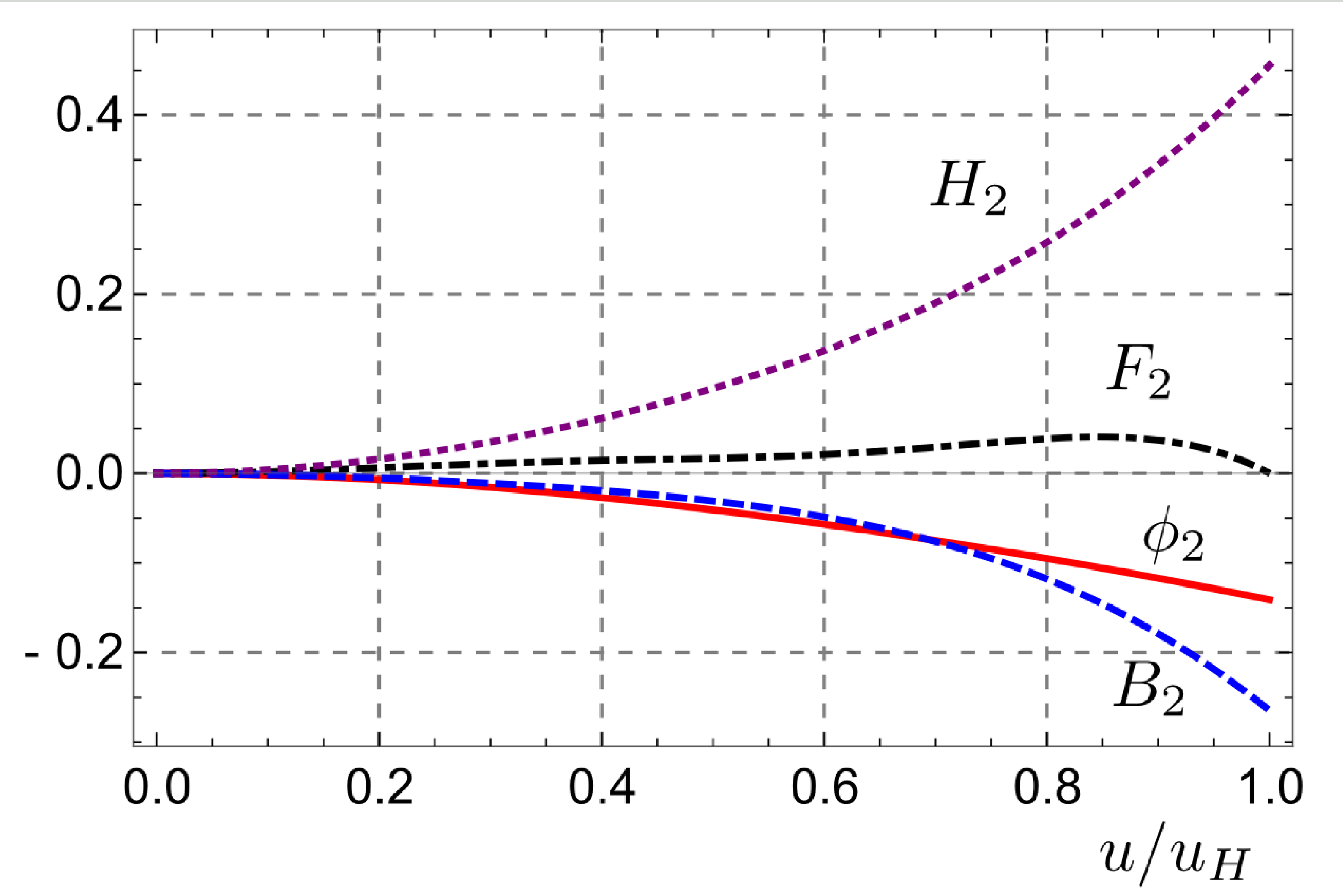

Figure 1: The metric functions at order $O(a^2)$. Here we have set $\lambda_{GB} = 0.2$.

- Thermodynamic quantities, such as the temperature T and the entropy density s can be easily computed via the standard expressions
$$T = -\frac{F'(u)\sqrt{B(u)}}{4\pi}\bigg|_{u=u_H}\,, \qquad s = \frac{\mathrm{Area}_{hor}}{4G_5\,\mathrm{Vol}_3}.$$

Holographic renormalization

- We use the recursive Hamilton-Jacobi holographic renormalization method that was introduced in [4] to compute the 1-point function of the boundary stress tensor associated to our gravitational system and we found a general expression, valid to first order in λ_{GB}. Specializing to our solution, up to order $O(a^2, \lambda_{GB})$, the components of the stress tensor read
$$\langle T_{tt}\rangle = 3\pi^4T^4\left[1 + \frac{1}{12\pi^2}\left(\frac{a}{T}\right)^2 + \left(\frac{3}{2} + \frac{1}{24\pi^2}\left(\frac{a}{T}\right)^2\right)\lambda_{GB}\right] + O(a^4, \lambda_{GB}^2)$$
$$\langle T_{xx}\rangle = \langle T_{yy}\rangle = \pi^4T^4\left[1 + \frac{1}{4\pi^2}\left(\frac{a}{T}\right)^2 + \left(\frac{3}{2} + \frac{1}{8\pi^2}\left(\frac{a}{T}\right)^2\right)\lambda_{GB}\right] + O(a^4, \lambda_{GB}^2)$$
$$\langle T_{zz}\rangle = \pi^4T^4\left[1 - \frac{1}{4\pi^2}\left(\frac{a}{T}\right)^2 + \left(\frac{3}{2} - \frac{1}{8\pi^2}\left(\frac{a}{T}\right)^2\right)\lambda_{GB}\right] + O(a^4, \lambda_{GB}^2)\,.$$
- These quantities correspond to the energy density and pressures of the dual gauge theory
$$E = \frac{N^2}{8\pi^2}\langle T_{tt}\rangle\,, \qquad P_\perp = \frac{N^2}{8\pi^2}\langle T_{xx}\rangle\,, \qquad P_\parallel = \frac{N^2}{8\pi^2}\langle T_{zz}\rangle\,,$$
with $P_\perp$ and $P_\parallel$ the pressures along the transverse plane and the longitudinal direction, respectively.
- At this order in a, the stress tensor is traceless, i.e., there is no conformal anomaly
$$\langle T^i_i\rangle = O(a^4, \lambda_{GB}^2)\,.$$

Shear viscosity

- We compute the shear viscosity over entropy ratio via membrane paradigm method [5] and we found, for the x-z component,
$$\frac{\eta_\parallel}{s} = \frac{1-4\lambda_{GB}}{4\pi} + \frac{B_0}{32\pi^3}G(\lambda_{GB})\left(\frac{a}{T}\right)^2 + O(a^4)\,,$$
where
$$G(\lambda_{GB}) = -1 + 2\lambda_{GB}\left(\frac{8\lambda_{GB}}{12\lambda_{GB}-3} + 1\right) + \sqrt{1-4\lambda_{GB}}$$
$$+ \sqrt{\lambda_{GB}}\log\left(\frac{1+2\sqrt{\lambda_{GB}}}{1-2\sqrt{\lambda_{GB}}}\right) + \log\left(\frac{\sqrt{1-4\lambda_{GB}} - 1 + 4\lambda_{GB}}{8\lambda_{GB}}\right).$$

Conclusions

- We found a black brane solution (perturbative in a, but fully nonperturbative in λ_{GB}) for AdS-gravity in five dimensions coupled to an axion-dilaton field with the inclusion of the Gauss-Bonnet term.
- One of our main concerns has been to carry out holographic renormalization and compute the 1-point function of the boundary stress tensor associated to our gravitational theory. We found a very general expression, valid to first order in the GB coupling.
- The shear viscosity over entropy density ratio violates the KSS bound [6], as expected from previous works [7, 8].
- Presumably numerical methods would have to be employed to extend our analysis to order $O(a^4)$. At this order probably we will have the presence of a conformal anomaly, as it happened in [3], where a rich phase diagram was discovered, with, in particular, the presence of instabilities that might turn out to be useful in understanding the fast thermalization time of the QGP.
- It would be interesting study other physical observables besides the shear viscosity, such as the energy loss via dragging and quenching, the quark-antiquark screening and the production of thermal photons.

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